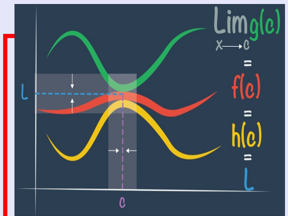
Math 261

Fall 2022

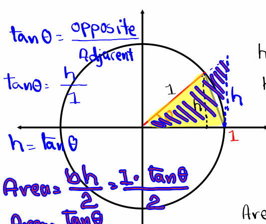
Lecture 10



$\tan\theta = \frac{\text{opposite}}{\text{Adjacent}}$

$\tan\theta = \frac{h}{1}$

$h = \tan\theta$



$\text{Area} = \frac{bh}{2} = \frac{1 \cdot \tan\theta}{2}$

$\text{Area} = \frac{\tan\theta}{2}$

$h = ?$ $\quad \sin\theta = \frac{h}{1}$

$h = \sin\theta$

$\text{Area} = \frac{bh}{2} = \frac{1 \cdot \sin\theta}{2}$

$\boxed{\text{Area} = \frac{\sin\theta}{2}}$

Area of the Sector

$A = \frac{r^2\theta}{2} = \frac{1^2 \cdot \theta}{2} = \frac{\theta}{2}$

Yellow Area < Area of Sector

$$\frac{\sin\theta}{2} < \frac{\theta}{2}$$

Yellow Area < Area of Sector < Area of new Triangle

$$\frac{\sin\theta}{2} < \frac{\theta}{2} < \frac{\tan\theta}{2}$$

Multiply by 2

$$\sin\theta < \theta < \tan\theta$$

Divide by $\sin\theta > 0$ in QI & QII $\quad \frac{\frac{\sin\theta}{\cos\theta}}{\sin\theta}$

$$\frac{\sin\theta}{\sin\theta} < \frac{\theta}{\sin\theta} < \frac{\tan\theta}{\sin\theta}$$

$$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$$

$\lim_{\theta \to 0} 1 = 1$, $\lim_{\theta \to 0} \frac{1}{\cos\theta} = \frac{1}{\cos 0} = \frac{1}{1} = 1$

By S.T. $\quad \lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

Find $\lim_{x \to 1} \frac{\sin(x-1)}{x^2-1} = \frac{\sin(1-1)}{1^2-1} = \frac{\sin 0}{0} = \frac{0}{0}$ I.F.

$$\lim_{x \to 1} \frac{\sin(x-1)}{x^2-1} = \lim_{x \to 1} \frac{\sin(x-1)}{(x+1)(x-1)} = \lim_{x \to 1} \left[\frac{1}{x+1} \cdot \frac{\sin(x-1)}{x-1} \right]$$

$$= \lim_{x \to 1} \frac{1}{x+1} \cdot \lim_{x \to 1} \frac{\sin(x-1)}{x-1}$$

$$\lim_{h \to 0} \frac{\sin h}{h} = 1$$

$$= \frac{1}{1+1} \cdot 1 = \boxed{\frac{1}{2}}$$

Find $\lim_{x \to 0} \frac{\sin 3x}{\sin 5x} = \frac{\sin 0}{\sin 0} = \frac{0}{0}$ I.F.

$$\lim_{x \to 0} \frac{\sin 3x}{\sin 5x} = \lim_{x \to 0} \frac{\frac{3 \sin 3x}{3x}}{\frac{5 \sin 5x}{5x}} = \frac{3}{5} \lim_{x \to 0} \frac{\frac{\sin 3x}{3x}}{\frac{\sin 5x}{5x}}$$

$$= \frac{3}{5} \cdot \frac{\lim_{x \to 0} \frac{\sin 3x}{3x}}{\lim_{x \to 0} \frac{\sin 5x}{5x}} = \frac{3}{5} \cdot \frac{1}{1} = \boxed{\frac{3}{5}}$$

Evaluate

$$\lim_{x\to 0} \frac{\sin 3x \sin 5x}{15x^2} = \frac{\sin 0 \cdot \sin 0}{15(0)^2} = \frac{0}{0} \text{ I.F.}$$

Recall $\lim_{h\to 0} \frac{\sin h}{h} = 1$

$$\lim_{x\to 0} \frac{\sin 3x \cdot \sin 5x}{15x^2} = \lim_{x\to 0} \left[\frac{\sin 3x}{3x} \cdot \frac{\sin 5x}{5x}\right]$$

$$= \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \lim_{x\to 0} \frac{\sin 5x}{5x}$$

$$= 1 \cdot 1 = \boxed{1}$$

Evaluate $\lim_{h\to 0} \frac{\tan 6h}{\sin 3h} = \frac{0}{0}$ I.F.

Recall $\lim_{h\to 0} \frac{\sin h}{h} = 1$

$$\lim_{h\to 0} \frac{\tan 6h}{\sin 3h} = \lim_{h\to 0} \frac{\frac{\sin 6h}{\cos 6h}}{\sin 3h} = \lim_{h\to 0} \frac{\sin 6h}{\sin 3h \cos 6h}$$

Recall $\sin 2A = 2\sin A \cos A$

$\sin 2(3h) = 2\sin 3h \cos 3h$

Recall $\cos 0 = 1$

$$= \lim_{h\to 0} \frac{2\cancel{\sin 3h}\cos 3h}{\cancel{\sin 3h}\cos 6h}$$

$$= \lim_{h\to 0} \frac{2\cos 3h}{\cos 6h}$$

$$= \frac{2 \cdot \cos 0}{\cos 0} = \boxed{2}$$

Find $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} = \frac{1-\cos 0}{0} = \frac{1-1}{0} = \frac{0}{0}$ I.F.

Recall
$\lim_{h\to 0} \frac{\sin h}{h} = 1$

$1-\cos^2\theta = \sin^2\theta$

$\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta}$

$= \lim_{\theta\to 0} \frac{1-\cos\theta}{\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta}$

$= \lim_{\theta\to 0} \frac{1-\cos^2\theta}{\theta(1+\cos\theta)} = \lim_{\theta\to 0} \frac{\sin^2\theta}{\theta(1+\cos\theta)}$

$= \lim_{\theta\to 0} \left[\frac{\sin\theta}{\theta} \cdot \frac{\sin\theta}{1+\cos\theta}\right]$

$= \underbrace{\lim_{\theta\to 0} \frac{\sin\theta}{\theta}}_{1} \cdot \lim_{\theta\to 0} \frac{\sin\theta}{1+\cos\theta}$

$\boxed{\lim_{h\to 0} \frac{1-\cosh}{h} = 0}$

$= 1 \cdot \frac{\sin 0}{1+\cos 0}$

$= 1 \cdot \frac{0}{2} = 1 \cdot 0 = \boxed{0}$

Evaluate $\lim_{\theta\to 0} \frac{1-\cos\theta}{\sin 3\theta}$

$= \frac{1-\cos 0}{\sin 0} = \frac{0}{0}$ I.F.

Recall
$\lim_{h\to 0} \frac{\sin h}{h} = 1$

$\lim_{h\to 0} \frac{1-\cos h}{h} = 0$

$\lim_{\theta\to 0} \frac{1-\cos\theta}{\sin 3\theta} = \lim_{\theta\to 0} \frac{\frac{1-\cos\theta}{\theta}}{3\frac{\sin 3\theta}{3\theta}}$

$= \frac{1}{3} \cdot \frac{\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta}}{\lim_{\theta\to 0} \frac{\sin 3\theta}{3\theta}} = \frac{1}{3} \cdot \frac{0}{1}$

$= \frac{1}{3} \cdot 0 = \boxed{0}$

Evaluate $\lim_{x \to 2} \dfrac{\sin(x-2)}{x^3-8} = \dfrac{0}{0}$ I.F.

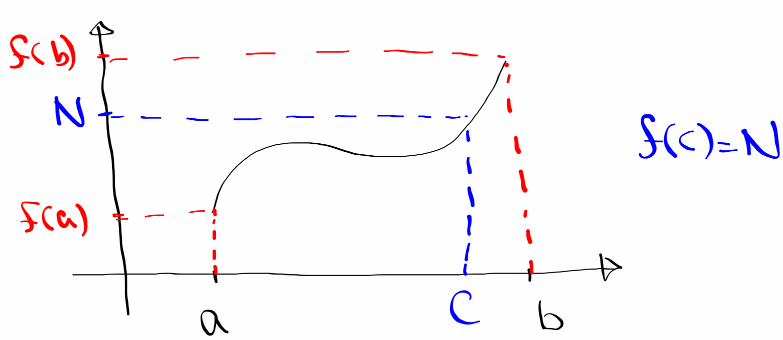
$$= \lim_{x \to 2} \frac{\sin(x-2)}{(x-2)(x^2+2x+4)}$$

$$= \lim_{x \to 2} \frac{\sin(x-2)}{x-2} \cdot \lim_{x \to 2} \frac{1}{x^2+2x+4}$$

$$= 1 \cdot \frac{1}{2^2+2(2)+4} = \boxed{\frac{1}{12}}$$

Intermediate Value Theorem:

Suppose $f(x)$ is a cont. function on a closed Interval $[a,b]$, and N is a number between $f(a)$ & $f(b)$ where $f(a) \neq f(b)$, there exist a number c in (a,b) such that $f(c)=N$.



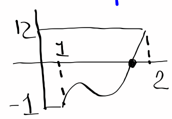
Show that $\underbrace{4x^3-6x^2+3x-2}_{f(x)}=0$

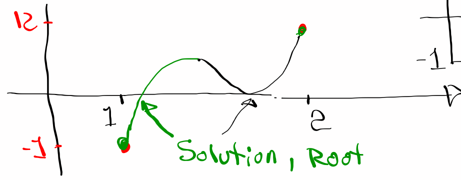
has a solution in $[1,2]$

Polynomial
Cont.
everywhere

$f(1)=4(1)^3-6(1)^2+3(1)-2=-1$

$f(2)=4(2)^3-6(2)^2+3(2)-2=12$





$f(1)<f(c)<f(2)$

$-1<0<12$

by I.V.T.
there is a
c in $(1,2)$
such that
$f(c)=0$

$x^2+5x-6=0$

Is 1 a Solution?

Consider
$[0,2]$